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**Branes on the Brane**H. Lü<sup>†1</sup> and C.N. Pope<sup>‡2</sup><sup>†</sup>*Department of Physics and Astronomy**University of Pennsylvania, Philadelphia, Pennsylvania 19104*<sup>‡</sup>*Center for Theoretical Physics**Texas A&M University, College Station, Texas 77843***ABSTRACT**

We show that four-dimensional  $N = 2$  ungauged Einstein-Maxwell supergravity can be embedded on the Randall-Sundrum 3-brane, as a consistent Kaluza-Klein reduction of five-dimensional  $N = 4$  gauged supergravity. In particular, this allows us to describe four-dimensional Reissner-Nordström black holes within the Randall-Sundrum scenario. Using earlier results on the embedding of five-dimensional  $N = 4$  gauged supergravity in ten dimensions, we can then describe the four-dimensional Einstein-Maxwell supergravity on the 3-brane, and its solutions, from a type IIB viewpoint. We also show that the minimal ungauged supergravities in  $D = 5$  and  $D = 6$  can be consistently embedded in the half-maximally supersymmetric gauged supergravities in  $D = 6$  and  $D = 7$  respectively. These allow us to construct solutions including BPS black holes and strings living in “Randall-Sundrum 4-branes,” and BPS self-dual strings living in “Randall-Sundrum 5-branes.” We can also lift the embeddings to ten-dimensional massive type IIA and  $D = 11$  supergravity respectively. In particular, we obtain a solution describing the self-dual string living in the world-volume of an M5-brane, which can be viewed as an open membrane ending on the M5-brane.

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# 1 Introduction

The Randall-Sundrum II scenario [1] has the intriguing feature that despite the existence of a non-compact fifth dimension, gravity is localised on the four-dimensional world-volume of the embedded 3-brane wall. This can also be understood from a Kaluza-Klein perspective, since one can consistently replace the Minkowski metric of the 3-brane world-volume by any Ricci-flat metric (see, for example, [2, 3, 4]), implying that pure Einstein gravity is certainly contained within the four-dimensional theory:

$$ds_5^2 = e^{-2k|z|} g_{\mu\nu} dx^\mu dx^\nu + dz^2. \quad (1)$$

(The constant  $k$  will be taken to be positive throughout this paper, since it is required for the trapping of gravity.) It is then natural to ask what kind of theory of gravity it is that resides on the 3-brane. Clearly the answer to this question depends upon the nature of the five-dimensional theory that is used in implementing the Randall-Sundrum scenario.

Naively, one might expect that if the five-dimensional theory is taken to be a gauged supergravity, then the theory on the 3-brane wall would be an ungauged supergravity with the same degree of supersymmetry, since the AdS spacetime preserves all supersymmetry. However, unlike an ordinary Kaluza-Klein reduction on a circle, the fifth direction  $z$  in (1) is not translationally invariant, and so in particular there will be no gauge-invariant massless Kaluza-Klein vector arising from the reduction.<sup>1</sup> This observation is already sufficient to show that the reduced theory in four dimensions cannot have as much supersymmetry as the five-dimensional gauged theory.

The situation can be clarified by looking at the Killing spinors in an AdS background, which in horospherical coordinates is given by  $ds^2 = e^{-2kz} dx^\mu dx_\mu + dz^2$ . The Killing spinors are of two kinds, given by [6]

$$\epsilon_+ = e^{-\frac{1}{2}kz} \epsilon_+^0, \quad \epsilon_- = \left( e^{\frac{1}{2}kz} - k e^{-\frac{1}{2}kz} x^\mu \Gamma_\mu \right) \epsilon_-^0, \quad (2)$$

where  $\epsilon_\pm^0$  are arbitrary constant spinors satisfying  $\Gamma_z \epsilon_\pm^0 = \pm \epsilon_\pm^0$ . In the Randall-Sundrum model, where the spacetime is taken to be symmetric about  $z = 0$ , one replaces  $z$  by  $|z|$  in (2):

$$\epsilon_+ = e^{-\frac{1}{2}k|z|} \epsilon_+^0, \quad \epsilon_- = \left( e^{\frac{1}{2}k|z|} - k e^{-\frac{1}{2}k|z|} x^\mu \Gamma_\mu \right) \epsilon_-^0, \quad (3)$$

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<sup>1</sup>It was recently observed in [5] that if one nevertheless writes a “standard” reduction ansatz including a Kaluza-Klein vector, then the inevitable lack of gauge invariance does not manifest itself until beyond the linearised level.

Clearly half of the Killing spinors, namely  $\epsilon_+$ , will be localised on the brane at  $z = 0$ , whilst the other half,  $\epsilon_-$  will not. This implies that the four-dimensional theory on the world-volume of the 3-brane has only half the supersymmetry of the five-dimensional gauged supergravity.

Thus if one considers Randall-Sundrum II in the framework of five-dimensional simple ( $N = 2$ ) gauged supergravity, the world-volume theory will then be four-dimensional simple ( $N = 1$ ) supergravity, whose bosonic sector comprises only the metric. We can also view this as a consistent Kaluza-Klein reduction of  $D = 5$ ,  $N = 2$  gauged supergravity to  $D = 4$ ,  $N = 1$  ungauged supergravity.

Four-dimensional  $N = 1$  supergravity does not support any BPS  $p$ -branes. The next simplest example in  $D = 4$  is  $N = 2$  Einstein-Maxwell supergravity, which admits the well-known Reissner-Nordström black hole. Following the previous argument, we would expect that it can be embedded, within the Randall-Sundrum picture, in  $N = 4$  gauged supergravity in  $D = 5$ . In section 2, we show that it is indeed the case, and so we can construct a Reissner-Nordström black hole in the world-volume of the 3-brane wall. The  $N = 4$  gauged supergravity itself can be obtained from a consistent 5-sphere reduction of type IIB supergravity [7], and so this provides a consistent embedding of Einstein-Maxwell supergravity in the world-volume of the ten-dimensional D3-brane. The resulting configuration can be viewed as an open string ending on the D3-brane.

In section 3 we generalise the procedure, to show that six-dimensional ungauged simple supergravity can be obtained as a consistent reduction of seven-dimensional  $N = 2$  gauged supergravity. This allows us to construct a self-dual string in the world-volume of the 5-brane wall of the seven-dimensional gauged theory. This theory itself can be obtained as a consistent 4-sphere reduction of  $D = 11$  supergravity [8, 9], and so the self-dual string can be lifted to eleven dimensions. The resulting configuration describes a self-dual string living in the M5-brane, which can be viewed as an open membrane ending on the M5-brane. In a similar vein, we also show that five-dimensional ungauged  $N = 2$  supergravity can be obtained as a consistent reduction of six-dimensional gauged  $N = 2$  supergravity. This allows us to obtain solutions for Reissner-Nordström black holes and strings living in the world-volume of the 4-brane wall. Since the six-dimensional gauged supergravity can itself be obtained as a local  $S^4$  reduction from the massive type IIA theory [10], we can also view these solutions as living in the world-volume of the D4/D8-brane system.

## 2 Reissner-Nordström black holes in Randall-Sundrum

## 2.1 Einstein-Maxwell supergravity via Randall-Sundrum

Here, we show that we can obtain ungauged four-dimensional Maxwell-Einstein ( $N = 2$ ) supergravity as a consistent Kaluza-Klein reduction of gauged five-dimensional  $N = 4$  supergravity, within a Randall-Sundrum type of framework. The bosonic sector of the five-dimensional theory comprises the metric, a dilatonic scalar  $\phi$ , the  $SU(2)$  Yang-Mills potentials  $A_{(1)}^i$ , a  $U(1)$  gauge potential  $B_{(1)}$ , and two 2-form potentials  $A_{(2)}^\alpha$  which transform as a charged doublet under the  $U(1)$ . The Lagrangian [11], expressed in the language of differential forms that we shall use here, is given by [7]

$$\begin{aligned}\mathcal{L}_5 = & R \tilde{*} \mathbf{1} - \frac{1}{2} \tilde{*} d\phi \wedge d\phi - \frac{1}{2} X^4 \tilde{*} G_{(2)} \wedge G_{(2)} - \frac{1}{2} X^{-2} (\tilde{*} F_{(2)}^i \wedge F_{(2)}^i + \tilde{*} A_{(2)}^\alpha \wedge A_{(2)}^\alpha) \\ & + \frac{1}{2g} \epsilon_{\alpha\beta} A_{(2)}^\alpha \wedge dA_{(2)}^\beta - \frac{1}{2} A_{(2)}^\alpha \wedge A_{(2)}^\alpha \wedge B_{(1)} - \frac{1}{2} F_{(2)}^i \wedge F_{(2)}^i \wedge B_{(1)} \\ & + 4g^2 (X^2 + 2X^{-1}) \tilde{*} \mathbf{1},\end{aligned}\tag{4}$$

where  $X = e^{-\frac{1}{\sqrt{6}}\phi}$ ,  $F_{(2)}^i = dA_{(1)}^i + \frac{1}{\sqrt{2}}g \epsilon^{ijk} A_{(1)}^j \wedge A_{(1)}^k$  and  $G_{(2)} = dB_{(1)}$ , and  $\tilde{*}$  denotes the five-dimensional Hodge dual. It is useful to adopt a complex notation for the two 2-form potentials, by defining

$$A_{(2)} \equiv A_{(2)}^1 + i A_{(2)}^2.\tag{5}$$

Our Kaluza-Klein reduction ansatz involves setting the fields  $\phi$ ,  $A_{(1)}^i$  and  $B_{(1)}$  to zero, with the remaining metric and 2-form potentials given by

$$\begin{aligned}ds_5^2 &= e^{-2k|z|} ds_4^2 + dz^2, \\ A_{(2)} &= \frac{1}{\sqrt{2}} e^{-k|z|} (F_{(2)} - i * F_{(2)}),\end{aligned}\tag{6}$$

where  $ds_4^2$  is the metric and  $F_{(2)}$  is the Maxwell field of the four-dimensional  $N = 2$  supergravity, and  $*$  denotes the Hodge dual in the four-dimensional metric.

To show that this ansatz gives a consistent reduction to four dimensions, we note from (4) that the five-dimensional equations of motion are [7]

$$\begin{aligned}d(X^{-1} \tilde{*} dX) &= \frac{1}{3} X^4 \tilde{*} G_{(2)} \wedge G_{(2)} - \frac{1}{6} X^{-2} (\tilde{*} F_{(2)}^i \wedge F_{(2)}^i + \tilde{*} \bar{A}_{(2)} \wedge A_{(2)}) \\ &\quad - \frac{4}{3} g^2 (X^2 - X^{-1}) \tilde{*} \mathbf{1}, \\ d(X^4 \tilde{*} G_{(2)}) &= -\frac{1}{2} F_{(2)}^i \wedge F_{(2)}^i - \frac{1}{2} \bar{A}_{(2)} \wedge A_{(2)}, \\ d(X^{-2} \tilde{*} F_{(2)}^i) &= \sqrt{2} g \epsilon^{ijk} X^{-2} \tilde{*} F_{(2)}^j \wedge A_{(1)}^k - F_{(2)}^i \wedge G_{(2)}, \\ X^2 \tilde{*} F_{(3)} &= -i g A_{(2)}, \\ R_{MN} &= 3X^{-2} \partial_M X \partial_N X - \frac{4}{3} g^2 (X^2 + 2X^{-1}) g_{MN} \\ &\quad + \frac{1}{2} X^4 (G_M^P G_{NP} - \frac{1}{6} g_{MN} G_{(2)}^2) + \frac{1}{2} X^{-2} (F_M^i F_{NP}^i - \frac{1}{6} g_{MN} (F_{(2)}^i)^2) \\ &\quad + \frac{1}{2} X^{-2} (\bar{A}_{(M}^P A_{N)P} - \frac{1}{6} g_{MN} |A_{(2)}|^2),\end{aligned}\tag{7}$$

where

$$F_{(3)} = DA_{(2)} \equiv dA_{(2)} - \mathbf{i} g B_{(1)} \wedge A_{(2)}. \quad (8)$$

It follows from (6) that

$$F_{(3)} = -\frac{1}{\sqrt{2}} k \epsilon(z) e^{-k|z|} (F_{(2)} - \mathbf{i} *F_{(2)}) \wedge dz + \frac{1}{\sqrt{2}} e^{-k|z|} (dF_{(2)} - \mathbf{i} d*F_{(2)}), \quad (9)$$

where  $\epsilon(z) = \pm 1$  according to whether  $z > 0$  or  $z < 0$ . Thus the equation of motion for  $F_3$  implies first of all that

$$dF_{(2)} = 0, \quad d*F_{(2)} = 0, \quad (10)$$

and so then, after taking the Hodge dual of the remaining terms in (9), we find from (7) that

$$-\frac{1}{\sqrt{2}} k \epsilon(z) e^{-k|z|} (*F_{(2)} + \mathbf{i} F_{(2)}) = -\frac{1}{\sqrt{2}} \mathbf{i} g e^{-k|z|} (F_{(2)} - \mathbf{i} *F_{(2)}), \quad (11)$$

which is identically satisfied provided that

$$g = \begin{cases} +k, & z > 0, \\ -k, & z < 0. \end{cases} \quad (12)$$

Since  $k$  is always positive (to ensure the trapping of gravity), this means that the Yang-Mills gauge coupling constant  $g$  has opposite signs on the two sides of the domain wall. This requirement is the same as the one imposed in [12] for the continuity of the Killing spinors across the boundary. This implies, as emphasised in [13], that the Randall-Sundrum scenario cannot arise strictly within the standard five-dimensional gauged supergravity, where  $g$  is a fixed parameter. It has a completely natural explanation from a ten-dimensional viewpoint, where  $g$  arises as a constant of integration in the solution for an antisymmetric tensor, and the imposed  $Z_2$  symmetry in fact *requires* that the sign must change across the wall [13]. For convenience, however, we shall commonly treat the coupling constant  $g$  of the gauged supergravity as if its sign can be freely chosen to be opposite on opposite sides of the domain wall, with the understanding that this can be justified from the higher-dimensional viewpoint.<sup>2</sup>

The equations of motion for  $X$  and  $G_{(2)}$  are satisfied since for our ansatz

$$\bar{A}_{(2)} \wedge A_{(2)} = 0 \quad (13)$$

and  $\tilde{*}A_{(2)} = \mathbf{i} A_{(2)}$ . The only remaining non-trivial equation in (7) is the Einstein equation. In vielbein components, the non-vanishing components of the Ricci tensor for the metric

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<sup>2</sup>An alternative possibility, developed in [14, 12], is to introduce an auxiliary field in the supergravity theory, with the gauge coupling constant (or mass parameter) arising as an integration constant when the auxiliary field is eliminated.

ansatz

$$d\hat{s}^2 = e^{-2k|z|} ds^2 + dz^2 \quad (14)$$

are given by

$$\begin{aligned} \hat{R}_{ab} &= e^{2k|z|} R_{ab} - (D-1)k^2 \eta_{ab} + 2k\delta(z)\eta_{ab}, \\ \hat{R}_{zz} &= -(D-1)k^2 + 2k(D-1)\delta(z), \end{aligned} \quad (15)$$

where, for future reference, we have given the general expressions for a reduction from  $D$  to  $(D-1)$  dimensions. Substituting into the five-dimensional Einstein equations, we find that the “internal” ( $zz$ ) component is identically satisfied, whilst the lower-dimensional components imply  $k^2 = g^2$  (consistent with (12)), and

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{2}(F_{\mu\rho}F_{\nu}{}^{\rho} - \frac{1}{4}F^2g_{\mu\nu}), \quad (16)$$

where  $R_{\mu\nu}$  is the four-dimensional Ricci tensor. Thus we have shown that the ansatz (6), when substituted into the equations of motion for the five-dimensional  $N=2$  gauged supergravity, gives rise to the equations of motion (10) and (16) of four-dimensional Einstein-Maxwell supergravity.

The fact that the Kaluza-Klein reduction that we have performed here gives a consistent reduction of the five-dimensional equations of motion to  $D=4$  is somewhat non-trivial, bearing in mind that the five-dimensional fields in (6) are required to depend on the coordinate  $z$  of the fifth dimension. The manner in which the  $z$ -dependence matches in the five-dimensional field equations so that consistent four-dimensional equations of motion emerge is rather analogous to the situation in a non-trivial Kaluza-Klein sphere reduction, although in the present case the required “conspiracies” are rather more easily seen.

As in the original Randall-Sundrum model [1], an external delta-function source is needed at  $z=0$ , to compensate the delta function in the five-dimensional Ricci tensor given by (15) that results from having introduced the modulus signs in  $|z|$  in (6). Smooth gravity trapping solutions remain elusive, and may be incompatible with supersymmetry in  $D \geq 5$  [15, 16, 17]. Of course if we omitted the modulus signs, our ansatz (6) would satisfy the bulk supergravity equations everywhere.<sup>3</sup>

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<sup>3</sup>We should emphasise that from a purely mathematical point of view all the Kaluza-Klein reductions that we consider in this paper can be recast as fully exact consistent reductions, with no delta-function sources needed, if we omit the modulus signs on  $z$  everywhere. There would also then be no sign-reversal of the gauge-coupling constant  $g$  on passing through  $z=0$ . However, since our goal is to describe supergravity localised on the domain wall, it is appropriate here to introduce the modulus signs, and pay the price of needing delta-function sources.

One indication of the localisation of gravity in the usual Randall-Sundrum model is the occurrence of the exponential factor in the metric  $ds_5^2 = e^{-2k|z|} dx^\mu dx_\mu + dz^2$ , which falls off as one moves away from the wall. It is therefore satisfactory that we have found that this same exponential fall-off occurs for the complete reduction ansatz (6), which we derived purely on the basis of the requirement of consistency of the embedding.

In our derivation of the reduction ansatz we have concentrated on the bosonic sectors of the four-dimensional and five-dimensional supergravities. Since we have proved the consistency in the bosonic sector, and since we know that the background where the four-dimensional metric is flat admits Killing spinors (namely the  $\epsilon_+$  spinors given in (2)), it follows that there must exist a straightforward extension of our ansatz to include the fermions. In fact we can easily see that the exponential  $z$  dependence matches properly in all the equations, with the vielbein and gauge fields having  $e^{-k|z|}$  factors in the reduction, while the Killing spinors have  $e^{-\frac{1}{2}k|z|}$  factors.

## 2.2 Lifting the Einstein-Maxwell embedding to $D = 10$

The consistent Kaluza-Klein reduction ansatz giving the embedding of the five-dimensional  $N = 2$  gauged supergravity in type IIB supergravity was derived in [7]. The internal reduction manifold is a 5-sphere, which can conveniently be described as a foliation by  $S^3 \times S^1$  surfaces. In this description, the unit  $S^5$  is given by

$$d\Omega_5^2 = d\xi^2 + \sin^2 \xi d\tau^2 + \cos^2 \xi d\Omega_3^2, \quad (17)$$

where  $0 \leq \xi \leq \frac{1}{2}\pi$ ,  $0 \leq \tau < 2\pi$ , and  $d\Omega_3^2$  is the metric on the unit 3-sphere. The  $SU(2) \times U(1)$  gauge fields parameterise transitively-acting translations on this  $S^3 \times S^1$  group submanifold, whilst the dilaton  $\phi$  parametrises inhomogeneous distortions of the 5-sphere. Lifting our ansatz (6) to  $D = 10$  is rather simple, since the scalar and  $SU(2) \times U(1)$  gauge fields vanish. From [7], we therefore find that (6) lifts to  $D = 10$  to give

$$\begin{aligned} d\hat{s}_{10}^2 &= e^{-2k|z|} ds_4^2 + dz^2 + g^{-2} \left( d\xi^2 + \sin^2 \xi d\tau^2 + \cos^2 \xi d\Omega_3^2 \right), \\ \hat{H}_{(5)} &= 4g \epsilon_{(5)} + 4g^{-5} \sin \xi \cos^3 \xi d\xi \wedge d\tau \wedge \Omega_{(3)}, \\ \hat{A}_{(2)} &= -\frac{1}{2}g^{-1} \sin \xi e^{-k|z| - i\tau} (F_{(2)} - i * F_{(2)}), \end{aligned} \quad (18)$$

where hats denote ten-dimensional quantities,  $\epsilon_5 = e^{-4k|z|} \epsilon_4 \wedge dz$ , with  $\epsilon_4$  being the volume form of the four-dimensional metric  $ds_4^2$ , and  $\Omega_{(3)}$  is the volume form of the unit 3-sphere. The complex 2-form  $\hat{A}_{(2)}$  is defined by

$$\hat{A}_{(2)} = A_{(2)}^{\text{RR}} + i A_{(2)}^{\text{NS}}, \quad (19)$$

where  $A_{(2)}^{\text{RS}}$  and  $A_{(2)}^{\text{NS}}$  are the R-R and NS-NS 2-form potentials of the type IIB theory.  $\hat{H}_{(5)}$  is the self-dual 5-form of the type IIB theory, and the ten dimensional dilaton and axion are set to zero.

### 2.3 Reissner-Nordström black holes on the brane

Having shown that four-dimensional ungauged Einstein-Maxwell supergravity arises as a consistent reduction of  $N = 4$  gauged five-dimensional supergravity, which in turn is a consistent  $S^5$  reduction of type IIB supergravity, we can now take any solution of the four-dimensional theory and lift it back to  $D = 5$  and  $D = 10$ . Examples of particular interest are the BPS Reissner-Nordström black holes in four dimensions, given by

$$\begin{aligned} ds_4^2 &= -H^{-2} dt^2 + H^2 dy^i dy^i, \\ F_{(2)} &= 2dt \wedge dH^{-1}, \end{aligned} \quad (20)$$

where  $H(y^i)$  is any harmonic function in the transverse 3-space.<sup>4</sup> This solution lifts straightforwardly to  $D = 5$ , using the ansatz (6):

$$\begin{aligned} ds_5^2 &= e^{-2k|z|} (-H^{-2} dt^2 + H^2 dy^i dy^i) + dz^2, \\ A_{(2)} &= \sqrt{2} e^{-k|z|} (dt \wedge dH^{-1} + \tfrac{i}{2} \epsilon_{ijk} \partial_i H dy^j \wedge dy^k). \end{aligned} \quad (21)$$

The solution could be thought of as a string, from the viewpoint of the five-dimensional bulk theory. Thus the solution describes an open string ending on a D3-brane. After lifting further to type IIB supergravity, using (18), we obtain

$$\begin{aligned} d\hat{s}_{10}^2 &= e^{-2k|z|} (-H^{-2} dt^2 + H^2 dy^i dy^i) + dz^2 + g^{-2} (d\xi^2 + \sin^2 \xi d\tau^2 + \cos^2 \xi d\Omega_3^2), \\ \hat{H}_{(5)} &= 4g e^{-4k|z|} H^2 dt \wedge d^3 y \wedge dz + 4g^{-5} \sin \xi \cos^3 \xi d\xi \wedge d\tau \wedge \Omega_{(3)}, \\ \hat{A}_{(2)} &= -g^{-1} \sin \xi e^{-k|z| - i\tau} (dt \wedge dH^{-1} + \tfrac{i}{2} \epsilon_{ijk} \partial_i H dy^j \wedge dy^k), \end{aligned} \quad (22)$$

where  $k$  and  $g$  are related by (12).

## 3 Simple supergravities embedded in gauged supergravities

In this section we shall discuss several examples of the embedding of an ungauged simple supergravity in a gauged supergravity in one higher dimension. We already discussed the embedding of four-dimensional simple supergravity in the introduction. Now, we shall consider two further examples, which are of greater interest in the sense that they contain bosonic fields other than just the metric itself, and so they admit BPS brane solutions.

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<sup>4</sup>We could also, of course, consider magnetically-charge BPS black holes, and non-extremal black holes.



### 3.1 Embedding minimal $D = 6$ supergravity in $D = 7$ and $D = 11$

Here, we shall show how simple ungauged supergravity in  $D = 6$ , whose bosonic sector comprises the metric and a self-dual 3-form, can be embedded in seven-dimensional  $N = 2$  gauged  $SU(2)$  supergravity. The bosonic sector of the seven-dimensional theory [18] comprises the metric, a dilatonic scalar  $\phi$ , the  $SU(2)$  Yang-Mills gauge potentials  $A_{(1)}^i$ , and a 3-form potential  $A_{(3)}$ . In the language of differential forms, which we shall use here, it can be described by the Lagrangian [9]

$$\begin{aligned} \mathcal{L}_7 = & R \tilde{*} \mathbb{1} - \frac{1}{2} \tilde{*} d\phi \wedge d\phi - g^2 \left( \frac{1}{4} X^{-8} - 2X^{-3} - 2X^2 \right) \tilde{*} \mathbb{1} - \frac{1}{2} X^4 \tilde{*} F_{(4)} \wedge F_{(4)} \\ & - \frac{1}{2} X^{-2} \tilde{*} F_{(2)}^i \wedge F_{(2)}^i + \frac{1}{2} F_{(2)}^i \wedge F_{(2)}^i \wedge A_{(3)} - \frac{1}{2\sqrt{2}} g F_{(4)} \wedge A_{(3)} , \end{aligned} \quad (23)$$

where  $X \equiv e^{-\phi/\sqrt{10}}$  and  $F_{(4)} = dA_{(3)}$ , together with the self-duality condition

$$X^4 \tilde{*} F_{(4)} = -\frac{1}{\sqrt{2}} g A_{(3)} + \frac{1}{2} \omega_{(3)} , \quad (24)$$

where  $\omega_{(3)} \equiv A_{(1)}^i \wedge F_{(2)}^i - \frac{1}{6} g \epsilon_{ijk} A_{(1)}^i \wedge A_{(1)}^j \wedge A_{(1)}^k$ . In (23) and (24),  $\tilde{*}$  denotes the Hodge dual in the seven-dimensional metric.

We find that we can consistently reduce this theory to  $D = 6$ , using an ansatz where  $A_{(1)}^i = 0$  and  $\phi = 0$ , together with

$$\begin{aligned} ds_7^2 &= e^{-2k|z|} ds_6^2 + dz^2 , \\ A_{(3)} &= \frac{1}{2k} e^{-2k|z|} F_3 , \end{aligned} \quad (25)$$

where  $F_{(3)}$  is a 3-form in the six-dimensional world-volume of the 4-brane wall. Thus  $F_{(4)} = \epsilon(z) e^{-2k|z|} F_{(3)} \wedge dz + 1/(2k) e^{-2k|z|} dF_{(3)}$  and so substituting into (24) we deduce that  $F_{(3)}$  must be either self-dual or anti-self dual, and  $dF_{(3)} = 0$ . Without loss of generality we shall take  $F_{(3)}$  to be self-dual, and so we have

$$g = \begin{cases} +2\sqrt{2}k , & z > 0 , \\ -2\sqrt{2}k , & z < 0 . \end{cases} \quad (26)$$

We find that all the seven-dimensional equations of motion (given, in our present notation, in [9]), are satisfied provided that  $ds_6^2$  and  $F_{(3)}$  satisfy the six-dimensional equations

$$R_{\mu\nu} = \frac{1}{4} F_{\mu\rho\sigma} F_{\nu}{}^{\rho\sigma} , \quad F_{(3)} = *F_{(3)} , \quad dF_{(3)} = 0 . \quad (27)$$

These are the bosonic field equations of six-dimensional minimal ungauged supergravity.

This embedding of the minimal ungauged six-dimensional supergravity can be lifted further, from  $D = 7$  to  $D = 11$ , by making use of the consistent  $S^4$  reduction of  $D =$

11 supergravity. The reduction to the maximal  $N = 4$  gauged theory was obtained in [8], and the reduction to the half-maximal  $N = 2$  gauged theory that we are considering here was constructed in [9]. In this embedding, the internal 4-sphere is described as a foliation of  $S^3$  surfaces, with the  $SU(2)$  Yang-Mills potentials parameterising transitively-acting translations on the  $S^3$  group submanifold. The dilaton parameterises inhomogeneous deformations of the 4-sphere. Since both  $A_{(1)}^i$  and  $\phi$  vanish in our  $D = 7$  to  $D = 6$  reduction ansatz (25), the lifting to  $D = 11$  is quite simple. Using the results in [9], we obtain the following expressions for the eleven-dimensional fields  $d\hat{s}_{11}^2$  and  $\hat{A}_{(3)}$ :

$$\begin{aligned} d\hat{s}_{11}^2 &= e^{-2k|z|} ds_6^2 + dz^2 + 2g^{-2} \left( d\xi^2 + \cos^2 \xi d\Omega_3^2 \right), \\ \hat{A}_{(3)} &= \frac{1}{2k} \sin \xi e^{-2k|z|} F_3 + 2\sqrt{2} g^{-3} \sin \xi (2 + \cos^2 \xi) \Omega_{(3)}, \end{aligned} \quad (28)$$

where  $\Omega_{(3)}$  is the volume form of the unit 3-sphere, and again  $g$  is related to  $k$  by (26). Note that the last term just gives a standard contribution proportional to the  $S^4$  volume form  $\Omega_{(4)} = \cos^3 \xi d\xi \wedge \Omega_{(3)}$  in the field strength  $\hat{F}_{(4)} = d\hat{A}_{(3)}$ .

Using these results, we can embed any solution of minimal ungauged six-dimensional supergravity in a ‘‘Randall-Sundrum 5-brane wall’’ solution of seven-dimensional  $N = 2$  gauged supergravity, and then in turn we can embed this in  $D = 11$  supergravity. In particular, we can consider a BPS self-dual string solution living in the 5-brane wall. As a solution of the minimal six-dimensional supergravity, this is given by

$$\begin{aligned} ds_6^2 &= H^{-1} (-dt^2 + dx^2) + H dy^i dy^i, \\ F_{(3)} &= dt \wedge dx \wedge dH^{-1} - \frac{1}{6} \partial_i H \epsilon_{ijk\ell} dy^j \wedge dy^k \wedge dy^\ell, \end{aligned} \quad (29)$$

where  $H(y^i)$  is harmonic in the transverse space. Lifted to  $D = 7$  using the ansatz (25), this gives

$$\begin{aligned} ds_7^2 &= e^{-2k|z|} \left( H^{-1} (-dt^2 + dx^2) + H dy^i dy^i \right) + dz^2, \\ A_{(3)} &= \frac{1}{2k} e^{-2k|z|} \left( dt \wedge dx \wedge dH^{-1} - \frac{1}{6} \partial_i H \epsilon_{ijk\ell} dy^j \wedge dy^k \wedge dy^\ell \right), \end{aligned} \quad (30)$$

as an embedding of the self-dual string in a 5-brane wall solution of seven-dimensional gauged supergravity.

This solution can then be further lifted back to  $D = 11$  using (28), yielding

$$\begin{aligned} d\hat{s}_{11}^2 &= e^{-2k|z|} \left( H^{-1} (-dt^2 + dx^2) + H dy^i dy^i \right) + dz^2 + 2g^{-2} (d\xi^2 + \cos^2 \xi d\Omega_3^2), \\ \hat{A}_{(3)} &= \frac{1}{2k} \sin \xi e^{-2k|z|} \left( dt \wedge dx \wedge dH^{-1} - \frac{1}{6} \partial_i H \epsilon_{ijk\ell} dy^j \wedge dy^k \wedge dy^\ell \right) \\ &\quad + 2\sqrt{2} g^{-3} \sin \xi (2 + \cos^2 \xi) \Omega_{(3)}, \end{aligned} \quad (31)$$

where the relation between  $k$  and  $g$  is given in (26). The solution can be viewed as an open membrane ending on a M5-brane.

### 3.2 Embedding minimal $D = 5$ supergravity in $D = 6$ and $D = 10$

In this section, we shall show that minimal ungauged five-dimensional supergravity can be obtained as a consistent Kaluza-Klein reduction of  $N = 2$   $SU(2)$ -gauged supergravity in  $D = 6$ .

The bosonic fields in this theory comprise the metric, a dilaton  $\phi$ , a 2-form potential  $A_{(2)}$ , and a 1-form potential  $B_{(1)}$ , together with the gauge potentials  $A_{(1)}^i$  of  $SU(2)$  Yang-Mills. The bosonic Lagrangian [19], converted to the language of differential forms, is [10]

$$\begin{aligned} \mathcal{L}_6 = & R \tilde{*} \mathbf{1} - \frac{1}{2} \tilde{*} d\phi \wedge d\phi - g^2 \left( \frac{2}{9} X^{-6} - \frac{8}{3} X^{-2} - 2X^2 \right) \tilde{*} \mathbf{1} \\ & - \frac{1}{2} X^4 \tilde{*} F_{(3)} \wedge F_{(3)} - \frac{1}{2} X^{-2} \left( \tilde{*} G_{(2)} \wedge G_{(2)} + \tilde{*} F_{(2)}^i \wedge F_{(2)}^i \right) \\ & - A_{(2)} \wedge \left( \frac{1}{2} dB_{(1)} \wedge dB_{(1)} + \frac{1}{3} g A_{(2)} \wedge dB_{(1)} + \frac{2}{27} g^2 A_{(2)} \wedge A_{(2)} + \frac{1}{2} F_{(2)}^i \wedge F_{(2)}^i \right), \end{aligned} \quad (32)$$

where  $X \equiv e^{-\phi/(2\sqrt{2})}$ ,  $F_{(3)} = dA_{(2)}$ ,  $G_{(2)} = dB_{(1)} + \frac{2}{3} g A_{(2)}$ ,  $F_{(2)}^i = dA_{(1)}^i + \frac{1}{2} g \epsilon_{ijk} A_{(1)}^j \wedge A_{(1)}^k$ , and here  $\tilde{*}$  denotes the six-dimensional Hodge dual.

We find that the following Kaluza-Klein ansatz gives a consistent reduction to minimal five-dimensional ungauged supergravity. Firstly, we set the potentials  $A_{(1)}^i$  and  $B_{(1)}$  to zero, and also set  $\phi = 0$ . The remaining fields are then taken to be

$$\begin{aligned} ds_6^2 &= e^{-2k|z|} ds_5^2 + dz^2, \\ G_{(2)} &= \sqrt{\frac{2}{3}} e^{-k|z|} F_{(2)}, \\ A_{(2)} &= \sqrt{\frac{3}{2}} g^{-1} e^{-k|z|} F_{(2)}, \end{aligned} \quad (33)$$

and the gauge-coupling  $g$  is taken to be

$$g = \begin{cases} +\frac{3}{\sqrt{2}} k, & z > 0, \\ -\frac{3}{\sqrt{2}} k, & z < 0. \end{cases} \quad (34)$$

We find that all the six-dimensional equations of motion (given, in our notation, in [10]), are then satisfied, provided that the fields  $ds_5^2$  and  $F_{(2)}$  satisfy the equations of motion of ungauged minimal five-dimensional supergravity:

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} &= \frac{1}{2} (F_{\mu\rho} F_{\nu}{}^{\rho} - \frac{1}{4} F^2 g_{\mu\nu}), \\ d*F_{(2)} &= \frac{1}{\sqrt{3}} F_{(2)} \wedge F_{(2)}, \quad dF_{(2)} = 0. \end{aligned} \quad (35)$$

This embedding of minimal ungauged five-dimensional supergravity on a “Randall-Sundrum 4-brane wall” solution of  $N = 2$  gauged six-dimensional supergravity can be further lifted to  $D = 10$ , using the fact that the six-dimensional gauged theory can be obtained via a local  $S^4$  reduction from the massive type IIA theory [10]. Substituting the ansatz (35) into the reduction ansatz obtained in [10], we find

$$\begin{aligned} d\hat{s}_{10}^2 &= (\sin \xi)^{\frac{1}{12}} \left[ e^{-2k|z|} ds_5^2 + dz^2 + 2g^{-2} \left( d\xi^2 + \cos^2 \xi d\Omega_3^2 \right) \right], \\ \hat{F}_{(4)} &= (\sin \xi)^{1/3} \left[ \frac{20\sqrt{2}}{3} g^{-3} \cos^3 \xi d\xi \wedge \Omega_{(3)} \right. \\ &\quad \left. + \frac{1}{\sqrt{3}} g^{-1} g^{-2k|z|} *F_{(2)} \wedge (\sqrt{2} \cos \xi d\xi - g \sin \xi dz) \right], \end{aligned} \quad (36)$$

$$\begin{aligned} \hat{F}_{(3)} &= \frac{1}{\sqrt{3}} (\sin \xi)^{-1/3} g^{-1} e^{-k|z|} F_{(2)} \wedge (\sqrt{2} \cos \xi d\xi - g \sin \xi dz), \\ \hat{F}_{(2)} &= \frac{1}{\sqrt{3}} (\sin \xi)^{2/3} e^{-k|z|} F_{(2)}, \\ e^{\hat{\phi}} &= (\sin \xi)^{-5/6}, \end{aligned} \quad (37)$$

where  $d\hat{s}_{10}^2$ ,  $\hat{\phi}$ ,  $\hat{F}_{(2)}$ ,  $\hat{F}_{(3)}$  and  $\hat{F}_{(4)}$  are fields of the massive type IIA theory [20], in the notation used in [10].

In a similar fashion to the previous examples, here we can construct non-dilatonic black hole or string solutions in the five-dimensional ungauged supergravity, and lift them first to solutions in the Randall-Sundrum 4-brane wall of the six-dimensional gauged theory, and then lift these further to  $D = 10$ . In ten dimensions, the black holes or strings live in the intersection of a D4/D8-brane system.

## 4 Spacetime structure and the AdS horizon

A detailed analysis of the Schwarzschild black hole, embedded in the Randall-Sundrum 3-brane wall, was carried out in [3]. It was shown that it could be viewed as a black string living in the five-dimensional AdS spacetime. Interestingly, although gravity is “localised on the brane” the four-dimensional Schwarzschild black hole has a profound influence on the spacetime geometry even out at the AdS horizon at  $z = \pm\infty$ , and indeed scalar curvature invariants diverge there [3]. The solution is not expected to be stable against “pinching off,” and it was argued in [3] that this would happen near the AdS horizon, leading to a stable “black cigar.” In this section we shall present a directly parallel discussion for some of our examples.

In the examples that we have considered in this paper, we can take the  $p$ -brane solutions on the Randall-Sundrum wall to be supersymmetric BPS configurations. It is easily seen that, as in the Schwarzschild example above, scalar curvature invariants will then diverge on

the AdS horizon. Indeed, this can already be seen from (15), which shows that if the Ricci tensor in the  $(D - 1)$ -dimensional spacetime of the brane wall is non-vanishing, then the Ricci tensor of the  $D$ -dimensional bulk spacetime will diverge exponentially as  $|z| \rightarrow \infty$ . (This is the same degree of divergence as was encountered for curvature invariants in [3].) For completeness, it is useful to present the full Riemann curvature for the metric reduction ansatz (14). In the natural vielbein basis, the orthonormal components of the Riemann tensor  $\hat{R}_{ABCD}$  of the  $D$ -dimensional metric are related to the components  $R_{abcd}$  for the  $(D - 1)$ -dimensional metric by

$$\begin{aligned}\hat{R}_{abcd} &= e^{2k|z|} R_{abcd} - k^2 (\eta_{ac} \eta_{bd} - \eta_{ad} \eta_{bc}), \\ \hat{R}_{zazb} &= -k^2 \eta_{ab} + 2k \delta(z) \eta_{ab}.\end{aligned}\tag{38}$$

If we neglect the delta-function terms arising from the discontinuity on the brane at  $z = 0$ , we find that the scalar invariant formed from the square of the Riemann tensor is given by

$$\hat{R}_{ABCD} \hat{R}^{ABCD} = e^{4k|z|} R_{abcd} R^{abcd} - 4k^2 e^{2k|z|} R + 2D(D - 1) k^4,\tag{39}$$

where  $R$  is the  $(D - 1)$ -dimensional Ricci scalar. Thus we see that in general non-vanishing curvature in the  $(D - 1)$ -dimensional metric on the brane wall leads to exponentially-diverging curvature on the AdS horizon, just as in [3]. A non-singular solution was constructed previously for a pp-wave propagating in AdS spacetime; the resulting metric is the generalised Kaigorodov metric, which is homogeneous [21]. A detailed analysis of pp-waves in a Randall-Sundrum brane has been given in [22].

Following [3], one can study geodesic motion in the  $p$ -brane metrics living in the Randall-Sundrum walls. We shall first discuss the example of the BPS black holes living in the 3-brane embedded in AdS<sub>5</sub>. If we consider a single isotropic BPS black hole, then from (21) the five-dimensional metric will be

$$ds_5^2 = e^{-2k|z|} \left[ - \left(1 + \frac{Q}{r}\right)^{-2} dt^2 + \left(1 + \frac{Q}{r}\right)^2 (dr^2 + r^2 d\Omega_2^2) \right] + dz^2.\tag{40}$$

Solving for geodesics moving in the equatorial plane, one finds that all timelike geodesics have non-constant  $z$ , as do all null geodesics that are not merely null geodesics of the four-dimensional metric. In the half-space  $z > 0$  (which we may consider without loss of generality), we then have

$$e^{-kz} = \begin{cases} -a \sin k\lambda, & \text{timelike,} \\ -a k\lambda, & \text{null,} \end{cases}\tag{41}$$

where  $\lambda$  is an affine parameter and  $a$  is a constant. Associated with the  $\partial/\partial t$  and  $\partial/\partial\varphi$  Killing vectors we have the two first integrals

$$\frac{dt}{d\lambda} = E e^{2kz} \left(1 + \frac{Q}{r}\right)^2, \quad \frac{d\varphi}{d\lambda} = L e^{2kz} \left(1 + \frac{Q}{r}\right)^{-2} r^{-2}, \quad (42)$$

where  $E$  and  $L$  are constants. The radial equation is

$$\left(\frac{d\tilde{r}}{d\nu}\right)^2 + \left[\left(1 + \frac{\tilde{Q}}{\tilde{r}}\right)^{-2} + \left(1 + \frac{\tilde{Q}}{\tilde{r}}\right)^{-4} \frac{\tilde{L}^2}{\tilde{r}^2} - \tilde{E}^2\right] = 0, \quad (43)$$

where we have performed the analogous transformations of the affine parameter to those in [3], namely

$$\nu = \begin{cases} -\frac{1}{a^2 k} \cot k\lambda, & \text{timelike,} \\ -\frac{1}{a^2 k^2 \lambda}, & \text{null,} \end{cases} \quad (44)$$

and defined rescaled quantities

$$r = a \tilde{r}, \quad Q = a \tilde{Q}, \quad L = a^2 \tilde{L}, \quad E = a \tilde{E}. \quad (45)$$

As in [3], we have the somewhat intriguing result that after performing the redefinition (44) of affine parameter, the radial equation (43) has reduced to the standard radial equation for timelike geodesics in the four-dimensional metric.

From the above results, we see that geodesics reach the horizon at  $z = \infty$  after a finite affine parameter interval, namely for  $\lambda$  approaching 0 from below. It then follows from (39) that the curvature diverges on the AdS horizon if  $R_{abcd}$  is non-vanishing for the metric  $ds_4^2$  on the brane. This will happen, for example, for geodesics that remain at finite  $r$  (*i.e.* those describing bound-state orbits). On the other hand the geodesics that reach  $r = \infty$  will have  $e^{kz} \sim -1/(a k \lambda)$  and  $r \sim -1/(a k^2 \lambda)$ . The square of the four-dimensional Riemann tensor for the single isotropic black-hole metric  $ds_4^2$  is given by

$$R^{abcd} R_{abcd} = \frac{8Q^2 (Q^2 + 6r^2)}{H^8 r^8}, \quad H = 1 + \frac{Q}{r}, \quad (46)$$

which goes like  $48Q^2/r^6$  as  $r$  tends to infinity. Thus we find from (39) that as the geodesic approaches the AdS horizon, the square of the five-dimensional Riemann tensor goes like

$$\hat{R}^{ABCD} \hat{R}_{ABCD} \sim 40k^4 + 48Q^2 a^2 k^8 \lambda^2, \quad (47)$$

which therefore remains finite.

As in [3], we can settle the question of whether these geodesics are actually avoiding the curvature singularity on the horizon by looking at the components of the Riemann tensor

in an orthonormal frame parallelly propagated along a timelike geodesic. Such a geodesic, with  $L = 0$ , has tangent vector  $u^A$  given by

$$u = -(a^2 e^{2kz} - 1)^{1/2} dz - E dt + H^2 (E^2 - a^2 H^{-2})^{1/2} dr \quad (48)$$

A unit normal vector  $n^A$  that is parallelly propagated ( $u^B \nabla_B n^A = 0$ ) along the geodesic is given by

$$n = a^{-1} e^{-kz} (E^2 - a^2 H^{-2})^{1/2} dt - a^{-1} E e^{-kz} H^2 dr. \quad (49)$$

From (38) we then find that in the bulk,

$$\hat{R}_{ABCD} u^A n^B u^C n^D = e^{2kz} R_{abcd} u^a n^b u^c n^d + k^2. \quad (50)$$

Using the specific form of the Riemann tensor  $R_{abcd}$  for the single isotropic black hole then gives

$$\hat{R}_{ABCD} u^A n^B u^C n^D \sim \frac{2Q a k^2}{\lambda} \quad (51)$$

as  $\lambda$  tends to zero, showing that the solution really does have a curvature singularity.

All of the above discussion closely parallels the discussion for the Schwarzschild black hole in Randall-Sundrum in [3]. In that case, the fact that the solution is non-extremal means that it is liable to suffer a Gregory-Laflamme [23] type of instability, leading to the pinching-off of the five-dimensional black string to give a black cigar. Since we can consider instead an extremal BPS solution, the possibility of such an instability does not then arise. Thus in comparison to the Schwarzschild embedding, the situation here is close, but no cigar.

More generally, we can repeat the above analysis for the self-dual string solution in the 5-brane domain wall, and the black-hole and string solutions in the 4-brane domain wall. We shall just summarise the results here. All the isotropic solutions in the various dimensions that we have discussed in this paper take the form, in the bulk,

$$d\hat{s}_D^2 = e^{-2k|z|} \left( H^{-2\alpha} dx^\mu dx_\mu + H^{2\beta} (dr^2 + r^2 d\Omega_n^2) \right) + dz^2, \quad (52)$$

with  $H = 1 + Q/r^{n-1}$ , where we have

$$\begin{aligned} D = 5 : & \quad \alpha = 1, \quad \beta = 1, \quad n = 2, \quad \text{black hole}, \\ D = 6 : & \quad \alpha = 1, \quad \beta = \frac{1}{2}, \quad n = 3, \quad \text{black hole}, \\ D = 6 : & \quad \alpha = \frac{1}{2}, \quad \beta = 1, \quad n = 2, \quad \text{string}, \\ D = 7 : & \quad \alpha = \frac{1}{2}, \quad \beta = \frac{1}{2}, \quad n = 3, \quad \text{self-dual string}. \end{aligned} \quad (53)$$

For motion in the equatorial plane of the  $n$ -sphere, in the region  $z > 0$ , we shall have first integrals

$$\frac{dt}{d\lambda} = E e^{2kz} H^{2\alpha}, \quad \frac{d\varphi}{d\lambda} = L e^{2kz} H^{-2\beta} r^{-2}, \quad (54)$$

and the  $z$  equation will again give

$$e^{-kz} = \begin{cases} -a \sin k\lambda, & \text{timelike,} \\ -a k\lambda, & \text{null,} \end{cases} \quad (55)$$

We again have that as  $z$  approaches infinity,  $e^{kz} \sim -1/(a k \lambda)$  and  $r \sim -1/(a k^2 \lambda)$ . It is easily seen that in all the cases the square of the hatted Riemann tensor, given by (39), will again go to zero along a timelike geodesic, as the AdS horizon is reached. Differences emerge, however, if we now examine the components of the hatted Riemann tensor with respect to an orthonormal frame parallelly propagated along a timelike geodesic. The analogue of the normal vector  $n$  in (49) is

$$n = a^{-1} e^{-kz} (E^2 - a^2 H^{-2\alpha})^{1/2} dt - a^{-1} E e^{-kz} H^{\alpha+\beta} dr. \quad (56)$$

Calculating  $\hat{R}_{ABCD} u^A n^B u^C n^D$ , we now find

$$\hat{R}_{ABCD} u^A n^B u^C n^D = a^2 e^{4kz} R_{0101} \sim \lambda^{n-3}, \quad (57)$$

where  $R_{0101}$  denotes the vielbein component of the  $(D-1)$ -dimensional Riemann tensor with 0 being time and 1 being the  $r$  direction. Thus in those cases where the transverse space has dimension 4, so that the  $n$ -sphere has  $n = 3$ , we find that  $\hat{R}_{ABCD} u^A n^B u^C n^D$  remains finite rather than diverging, as  $\lambda$  tends to zero. This occurs in the case of black-holes living in the 4-brane in  $D = 6$ , and self-dual strings living in the 5-brane in  $D = 7$ . Although we have only exhibited one parallelly-propagated curvature component, it seems likely that the same feature will occur for all the components. This is because the  $\lambda^{n-3}$  dependence seen in (57) simply arises from the trade-off between the diverging  $e^{4kz}$  factor and the  $1/r^{n+1}$  fall-off of the curvature in the  $(D-1)$ -dimensional brane metric.

## 5 Discussion and conclusion

In this paper, we have shown that ungauged  $N = 2$  supergravity arises on the four-dimensional Randall-Sundrum 3-brane wall, obtained as a solution of  $N = 4$   $SU(2) \times U(1)$  gauged supergravity in five dimensions. The four-dimensional supergravity emerges through a Kaluza-Klein mechanism, as a consistent reduction from  $D = 5$ . Any solution of the four-dimensional  $N = 2$  theory can therefore be lifted back to five dimensions, and it acquires



an interpretation as a solution in the Randall-Sundrum wall. In particular, the  $N = 2$  supergravity is large enough to admit BPS black hole solutions, and so we can view these as living within the domain wall. Since the five-dimensional  $N = 4$  gauged supergravity can itself be embedded in the ten-dimensional type IIB theory, via a consistent  $S^5$  reduction [7], the solutions can thereby be lifted to  $D = 10$ .

We also considered the consistent embeddings of ungauged minimal supergravities in gauged supergravities for a variety of dimensions. The most immediately physically relevant example would be the embedding of four-dimensional simple supergravity in  $N = 2$  gauged five-dimensional supergravity. Since the metric is the only bosonic field in the four-dimensional supergravity, this example does not go beyond previous results, such as in [3], where Ricci-flat solutions such as the Schwarzschild black hole can be viewed as living on the 3-brane. In this paper we considered instead the minimal ungauged supergravities in  $D = 5$  and  $D = 6$ . The former contains gravity and a 2-form field strength in its bosonic sector, while the latter contains gravity and a self-dual 3-form. Both cases, therefore, admit BPS solutions. We showed that the  $D = 5$  minimal supergravity could be embedded in six-dimensional  $N = 2$   $SU(2)$ -gauged supergravity, whilst the  $D = 6$  minimal supergravity could be embedded in seven-dimensional  $N = 2$   $SU(2)$ -gauged supergravity. Thus we were able to describe higher-dimensional examples of strings and black holes living in “Randall-Sundrum 4-branes,” and self-dual strings living in “Randall-Sundrum 5-branes.” In each case, a further lifting can be performed, to  $D = 10$  massive IIA, and to  $D = 11$ , respectively. In particular, we obtain a solution describing a self-dual string living in the world-volume of an M5-brane. As was shown in [24], half-maximal gauged supergravities can also be embedded in singular warped spacetimes in  $D = 11$  or  $D = 10$ . The BPS black hole or string solutions we obtained in this paper can then live in the world-volumes of intersecting M-branes or D-branes in higher dimensions.

We concentrated on relatively simple examples of Kaluza-Klein domain-wall reductions in this paper, but the procedure can be generalised to more complicated cases, with larger gauged supergravities with more supersymmetries.

An analysis of the spacetime structures of the embedded solutions suggests that when the metric on the brane wall has non-vanishing curvature, there will in general be curvature singularities at the AdS horizons, far from the Randall-Sundrum wall. This phenomenon was studied in [3] for a Schwarzschild black hole. Since in that case the solution is non-supersymmetric, an instability is expected to set in near the horizon, leading to the degeneration of the five-dimensional black string to a black cigar. No such instability should occur in

our examples, if we take the solutions on the brane wall to be extremal BPS  $p$ -branes. The divergent curvature on the AdS horizons can be taken as an indication that strong-coupling effects are setting in, which would mean that the supergravity solution would no longer be trustworthy in the region near the horizon. However, the analysis that we have carried out should be valid on and near the domain wall itself, where the ungauged supergravity is localised.

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## References

- [1] L. Randall and R. Sundrum, *An alternative to compactification*, Phys. Rev. Lett. **83** (1999) 4690, hep-th/9906064.
- [2] D. Brecher and M.J. Perry, *Ricci-flat branes*, Nucl. Phys. **B566** (2000) 151, hep-th/9908018.
- [3] A. Chamblin, S.W. Hawking and H.S. Reall, *Brane world black holes*, Phys. Rev. **D61** (2000) 065007, hep-th/9909205.
- [4] I. Giannakis and H.C. Ren, *Recovery of the Schwarzschild metric in theories with localised gravity beyond linear order*, hep-th/0007053.
- [5] G. Kang and Y.S. Myung,  *$U(1)$  gauge field of the Kaluza-Klein theory in the presence of branes*, hep-th/0007197.
- [6] H. Lü, C.N. Pope and P.K. Townsend, *Domain walls from anti-de Sitter spacetime*, Phys. Lett. **B391** (1977) 39, hep-th/9607164.
- [7] H. Lü, C.N. Pope and T.A. Tran, *Five-dimensional  $N = 4$   $SU(2) \times U(1)$  gauged supergravity from type IIB*, Phys. Lett. **B475** (2000) 261, hep-th/9909203.
- [8] H. Nastase, D. Vaman and P. van Nieuwenhuizen, *Consistency of the  $AdS_7 \times S_4$  reduction and the origin of self-duality in odd dimensions*, Nucl. Phys. **B581** (2000) 179, hep-th/9911238.

- [9] H. Lü and C.N. Pope, *Exact embedding of  $N = 1, D = 7$  gauged supergravity in  $D = 11$* , Phys. Lett. **B467** (1999) 67, hep-th/9906168.
- [10] M. Cvetič, H. Lü and C.N. Pope, *Gauged six-dimensional supergravity from massive type IIA*, Phys. Rev. Lett. **83** (1999) 5226, hep-th/9906221.
- [11] M. Günaydin, L.J. Romans and N.P. Warner, *Compact and non-compact gauged supergravity theories in five dimensions*, Nucl. Phys. **B272** (1986) 598.
- [12] E. Bergshoeff, R. Kallosh and A. Van Proeyen, *Supersymmetry in singular spaces*, hep-th/0007044.
- [13] M.J. Duff, J.T. Liu and K.S. Stelle, *A supersymmetric type IIB Randall-Sundrum realization*, hep-th/0007120.
- [14] E. Bergshoeff, M. de Roo, M.B. Green, G. Papadopoulos and P.K. Townsend, *Duality of type II 7-branes and 8-branes*, Nucl. Phys. **B470** (1996) 113, hep-th/9601150.
- [15] R. Kallosh and A. Linde, *Supersymmetry and the brane world*, JHEP **0002**, 005 (2000), hep-th/0001071.
- [16] K. Behrndt and M. Cvetič, *Anti-de Sitter vacua of gauged supergravities with eight supercharges*, Phys. Rev. **D61** (2000) 101901, hep-th/0001159.
- [17] G.W. Gibbons and N.D. Lambert, *Domain walls and solitons in odd dimensions*, hep-th/0003197.
- [18] P.K. Townsend and P. van Nieuwenhuizen, *Gauged seven-dimensional supergravity*, Phys. Lett. **B125** (1983) 41.
- [19] L.J. Romans, *The  $F(4)$  gauged supergravity in six dimensions*, Nucl. Phys. **B269** (1986) 691.
- [20] L.J. Romans, *Massive  $N = 2a$  supergravity in ten dimensions*, Phys. Lett. **B169** (1986) 374.
- [21] M. Cvetič, H. Lu and C.N. Pope, *Spacetimes of boosted  $p$ -branes, and CFT in infinite-momentum frame*, Nucl. Phys. **B545** (1999) 309, hep-th/9810123.
- [22] A. Chamblin and G.W. Gibbons, *Supergravity on the brane*, Phys. Rev. Lett. **84** (2000) 1090, hep-th/9909130.

- [23] R. Gregory and R. Laflamme, Phys. Rev. Lett. **70** (1993) 2837, hep-th/9301052.
- [24] M. Cvetič, H. Lü and C.N. Pope, *Consistent warped-space Kaluza-Klein reductions, half-maximal gauged supergravities and  $CP^n$  constructions*, hep-th/0007109.